An interesting special case of such a discontinuity is that in which there is moving gas on one side and radiation on the other side. The flow pattern in this case is the following: In moving, the gas flows around a certain void occupied by radiation. It is easy to plot a flow past a wedge-shaped or cone-shaped void occupied by radiation.

If W is small but finite, the tangential discontinuity occupies, obviously, a region having the thickness

$$\delta \sim L\sqrt{W} \tag{12.4}$$

In this case we are dealing with a radiation boundary-layer, a theory for which easily can be constructed, and the equations for which assume a class of similarity solutions having the form:

$$\begin{aligned} u &= x^{\alpha}\bar{u}(\zeta) & v &= x^{\beta}\bar{v}(\xi) & p &= x^{\nu}\bar{p}(\xi) \\ \rho &\approx x^{\delta}\bar{\rho}(\zeta) & T &= x^{\mu}\bar{T}(\xi) & \xi &= x^{\nu}y \end{aligned} \tag{12.5}$$

If in Eqs. (12.1) the absorption coefficient $\bar{k} = C\cdot p^n T^m$, then

$$\bar{\alpha} = \nu + \tfrac{1}{2} \qquad \bar{\beta} = (\nu + 1)(2 - \tau) - \tfrac{1}{2} \qquad \bar{\gamma} = 4(2\nu + 1)$$

$$\bar{\delta} = 3(2\nu + 1) \qquad \bar{\mu} = 2\nu + 1$$

$$\bar{\alpha} = \alpha\tau = \alpha(m + 3n + \tfrac{3}{2})$$

The forementioned singularities in the case $W \ll 1$ follow directly from the circumstance that nearly black-body radiation exhibits a diffusion character, and for this reason the same phenomena occur that are found when we examine the dissipation processes both in ordinary fluid-dynamics and in magnetohydrodynamics (boundary layer, the "squeezing-out" phenomenon, etc.). In the case where W is small and $p_r \ll p$ the radiation boundary-layer is transformed into a thermal layer caused by the radiation flux. In this case the quantity $1/W$ can be regarded as the Péclet number, linked to the radiation's thermal conductivity, i.e., $1/W = Pe^* = Pr^*Re$, where Pr^* is the Prandtl number for the radiation, Re the ordinary Reynolds number. Since we have determined the parameter W in the case $\omega \gg 1$, we have

$$Pe^* = \frac{i\cdot\nu}{U\cdot c} \tag{12.6}$$

where i is the enthalpy of the radiating gas per unit volume, and U is the energy density of the radiation field.

Assuming the presence of recombination radiation, we evaluated the Pe^* numbers for sets of flight conditions analogous to the sets of conditions in the examples of Sec. 6—namely, for altitudes $H = 30$ km and $H = 40$ km, both with velocity $v = 1.5 \times 10^6$ cm/sec, and then for $H = 30$ km with velocity $v = 10^6$ cm/sec—and we arrived at the values $Pe^* = 1350$, 25, and 320, respectively.

The Reynolds numbers here range from 10^7 to 10^9 (the characteristic dimension is $L = 10^2$ cm). As a result of this we are able to infer that in the case in question, where $1 \ll Pe^* \ll Re$, we shall encounter a phenomenon of thermal boundary-layers with small Prandtl numbers.

The tangential discontinuity that has been examined for the case of an ideally absorbing medium can have meaning also for internal flows in which radiation is present in the flow core. The flow itself in this case will function as mirror. The question as to the effect exerted by nearly black-body radiation on the character of the boundary layer when the magnetic terms are taken into account was examined previously in Ref. 20.

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Reviewer's Comment

This paper constitutes a small monograph introducing the fundamental concepts of radiation transfer, their contribution to the formulation of the conservation principles in fluid dynamics, and a cursory survey of some of the aerodynamic applications in which radiation transfer is coupled with the other transport effects. Also, an important section is devoted to the estimation of the emissivity of high temperature air. With regard to these different parts, the following remarks may be made:

1. Fundamental Relations of Radiation Gasdynamics (Secs. 1–4)

The results presented in Secs. 1–4 are well established in the literature and were reviewed with particular clarity for the gasdynamics problem by Prokofiev.[1] It is not clear why the authors choose in Eq. (2.2) to introduce the volume force **F**, which seems to be an unnecessary restriction on the usual radiation pressure gradient $p_{,i}{}^{R}$. This distinction was discussed by Aller (Ref. 6 of the paper).

Also, the discussion in Sec. 4 of why radiation *forces* should be neglected seems laborious when compared with the elegant analysis due to Rocard (Ref. 2, p. 248). Finally, there was hardly a need to introduce new symbols for the optical length and the radiation-convection parameters (ω, W) when too many other symbols are already in use $(\tau, N_{Bu}, N_{Bo}, \Gamma$, etc.).

2. Emissivity of High Temperature Air (Secs. 5–6)

The high temperature air-radiation estimates are based on the same transition mechanisms as were used in similar estimates in the United States.[3, 4] (Breene, however, includes as an appreciable contributor the $[N^+ + O^+]$ free-free radiation, in addition to the de-ionization process considered by Zhigulev.) As for using Zhigulev's estimates (i.e., from the standpoint of the fluid dynamicist), it is regrettable that the paper offers only formulas for the emissivity of air and not the convenient graphs available from Refs. 3 and 4. A few check points taken from later sections of the paper seem to show estimated emissivity values slightly larger than those in Refs. 3 and 4.

In Sec. 6, estimates are presented of the radiation-convection ratio W for typical hypersonic flight conditions (Fig. 4). It seems that an error was introduced on this figure; L should probably read L' (Eq. 6.3), and $L = 10^2$ cm $= 1$ m. Better yet, the ordinate W could be replaced by W/L, m^{-1}, allowing for any value of L [Eq. (6.1)].

Note that this plot of three variables (W, velocity, altitude) can be arranged in three permutative forms using two variables as the coordinates and tracing lines of constant values for the third variable; the other two forms of Fig. 4 can be found in Ref. 5.

Both this paper and Ref. 5 lack generality in that they consider only the optically thin case, whereas the true radiation-dynamic parameter is $q_0{}^R/\rho_\infty u_\infty h_0$, where $q_0{}^R$ takes a different form if the gas is not optically thin. Reference 6 illustrates this point clearly.

Finally, it may be noted that nonequilibrium effects will upset the estimates of emissivity whenever the ambient density is not large enough to support chemical equilibrium behind the shock.[7] For cases of astronautical interest ($L < 1$ m), the right-hand half of Fig. 4 is not likely to be valid. Some recent estimates of these effects can be found in Ref. 8.

3. Applications to Hypersonic Flow Problems (Secs. 7–12)

In Secs. 7–12 several problems are considered for the first time. Since some of these have since been analyzed in greater detail, it would not be too profitable to analyze here what was meant to be essentially an order-of-magnitude appraisal, correct for the most part in its implications. Two exceptions are Sec. 10, which is a fine linearized treatment of the supersonic wedge in radiation fluid dynamics, and Sec. 8, which is a rather poor comparison between conductive and radiative heat transfer; the conductive heat transfer is *inversely* proportional to the thermal boundary-layer thickness δ_T [Eq. (8.1)], and therefore the radiation-conduction ratio should read $W\delta_R\delta_T$, or $WR^{3/2}$, as can be seen in Ref. 3 of the paper.

4. Conclusion

In conclusion, the paper under discussion had the great merit to put together, for the first time in logical sequence, the outline of an area of aerodynamics in which radiation is an appreciable energy contributor. The implications for current space programs are obvious, and much new work has been published in the last two years on this subject. A fresh review of this area of fluid dynamics would be a timely contribution.

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